IBO

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N06/5/MATHL/HP2/ENG/TZ0/XX

88067202

MATHEMATICS
HIGHER LEVEL
PAPER 2

Friday 3 November 2006 (morning)

2 hours

INSTRUCTIONS TO CANDIDATES

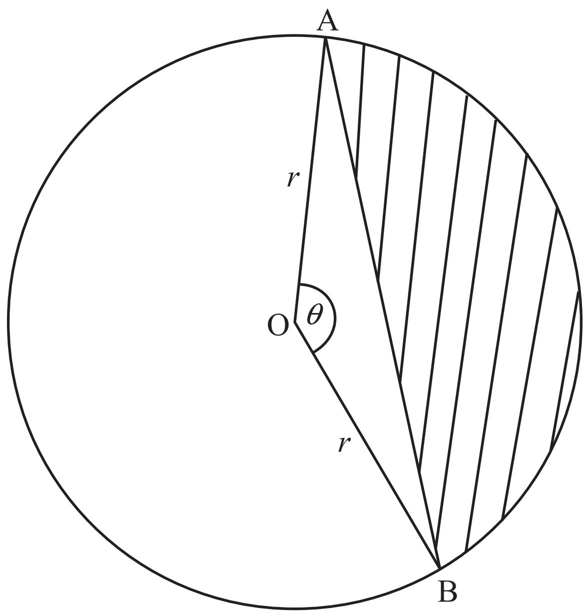
- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Total Mark: 24]*

Part A *[Maximum mark: 13]*

The following diagram shows a circle centre O, radius r. The angle $\mathrm{A\hat{O}B}$ at the centre of the circle is θ radians. The chord AB divides the circle into a minor segment (the shaded region) and a major segment.



(a) Show that the area of the minor segment is $\frac{1}{2}r^2(\theta - \sin\theta)$. *[4 marks]*

(b) Find the area of the major segment. *[3 marks]*

(c) Given that the ratio of the areas of the two segments is $2:3$, show that $\sin\theta = \theta - \frac{4\pi}{5}$. *[4 marks]*

(d) Hence find the value of θ. *[2 marks]*

(This question continues on the following page)

(Question 1 continued)

Part B *[Maximum mark: 11]*

(a) Use mathematical induction to prove that

$$(1)(1!)+(2)(2!)+(3)(3!)+...+(n)(n!)=(n+1)!-1 \text{ where } n\in\mathbb{Z}^{+}.$$ *[8 marks]*

(b) Find the minimum number of terms of the series for the sum to exceed 10^{9}. *[3 marks]*

2. *[Total Mark: 22]*

Part A *[Maximum mark: 12]*

A bag contains a very large number of ribbons. One quarter of the ribbons are yellow and the rest are blue. Ten ribbons are selected at random from the bag.

(a) Find the expected number of yellow ribbons selected. *[2 marks]*

(b) Find the probability that exactly six of these ribbons are yellow. *[2 marks]*

(c) Find the probability that at least two of these ribbons are yellow. *[3 marks]*

(d) Find the most likely number of yellow ribbons selected. *[4 marks]*

(e) What assumption have you made about the probability of selecting a yellow ribbon? *[1 mark]*

Part B *[Maximum mark: 10]*

The continuous random variable *X* has probability density function

$$f(x)=\begin{cases}\dfrac{x}{1+x^{2}}, & \text{for } 0\le x\le k\\ 0, & \text{otherwise.}\end{cases}$$

(a) Find the exact value of *k*. *[5 marks]*

(b) Find the mode of *X*. *[2 marks]*

(c) Calculate $\mathrm{P}(1\le X\le 2)$. *[3 marks]*

3. *[Total Mark: 28]*

Part A *[Maximum mark: 14]*

(a) The line l_1 passes through the point $\mathrm{A}(0, 1, 2)$ and is perpendicular to the plane $x - 4y - 3z = 0$. Find the Cartesian equations of l_1. *[2 marks]*

(b) The line l_2 is parallel to l_1 and passes through the point $\mathrm{P}(3, -8, -11)$. Find the vector equation of the line l_2. *[2 marks]*

(c) (i) The point Q is on the line l_1 such that $\overrightarrow{\mathrm{PQ}}$ is perpendicular to l_1 and l_2. Find the coordinates of Q.

(ii) Hence find the distance between l_1 and l_2. *[10 marks]*

Part B *[Maximum mark: 14]*

Consider the system of equations

$$\begin{aligned} x + 2y + kz &= 0 \\ x + 3y + z &= 3 \\ kx + 8y + 5z &= 6 \end{aligned}$$

(a) Find the set of values of k for which this system of equations has a **unique** solution. *[6 marks]*

(b) For each value of k that results in a **non-unique** solution, find the solution set. *[8 marks]*

4. *[Maximum mark: 26]*

The function f is defined by $f(x) = \dfrac{\ln x}{x^3}$, $x \geq 1$.

(a) Find $f'(x)$ and $f''(x)$, simplifying your answers. *[6 marks]*

(b) (i) Find the **exact** value of the x-coordinate of the maximum point and justify that this is a maximum.

(ii) Solve $f''(x) = 0$, and show that at this value of x, there is a point of inflexion on the graph of f.

(iii) Sketch the graph of f, indicating the maximum point and the point of inflexion. *[11 marks]*

The region enclosed by the x-axis, the graph of f and the line $x = 3$ is denoted by R.

(c) Find the volume of the solid of revolution obtained when R is rotated through 360° about the x-axis. *[3 marks]*

(d) Show that the area of R is $\dfrac{1}{18}(4 - \ln 3)$. *[6 marks]*

5. *[Maximum mark: 20]*

Let $y = \cos\theta + \mathrm{i}\sin\theta$.

(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}\theta} = \mathrm{i}y$.
[You may assume that for the purposes of differentiation and integration, i may be treated in the same way as a real constant.] *[3 marks]*

(b) **Hence** show, using integration, that $y = \mathrm{e}^{\mathrm{i}\theta}$. *[5 marks]*

(c) Use this result to deduce de Moivre's theorem. *[2 marks]*

(d) (i) Given that $\dfrac{\sin 6\theta}{\sin\theta} = a\cos^5\theta + b\cos^3\theta + c\cos\theta$, where $\sin\theta \neq 0$, use de Moivre's theorem with $n = 6$ to find the values of the constants a, b and c.

(ii) **Hence** deduce the value of $\lim\limits_{\theta \to 0} \dfrac{\sin 6\theta}{\sin\theta}$. *[10 marks]*